Design of Desirable Airplane Handling Qualities via Optimal Control

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The technique known as implicit model following allows desired closed-loop characteristics of a system to be included in an optimal control algorithm. Previous authors have used a model of the form $\dot{x} = Lx$, where x is the state vector. The optimal control algorithm then computes the feedback which makes the system response optimally close to the model. This paper extends this approach to include all desired handling qualities by using the model $\dot{x} = Lx + N\delta$, where δ is the vector of pilot commands. The algorithm which is derived computes both optimal feedback and optimal feedforward from δ to the controls. Algorithms are given for both sampled-data and continuous control. General guidelines for choosing L and N are presented. An example is given for the design of the landing approach control for a short takeoff and landing (STOL) airplane.

Nomenclature

A = square matrix of differential equation coefficients (airplane)
D = matrix of differential equation coefficients of control influence
F = optimal feedforward matrix
G_{xx} = square symmetric matrix which determines $x'G_{xx}x$ component of cost function
$G_{x\delta}$ = matrix which determines $x'G_{x\delta}\delta$ component of cost function
$G_{\delta\delta}$ = matrix which determines $\delta'G_{\delta\delta}\delta$ component of cost function
$\dot{h}$ = rate of change of altitude (positive downward)
I = identity matrix
J,J_c = cost function for discrete-time, continuous control, respectively
L = square matrix of differential equations coefficients (implicit model)
M = optimal feedback matrix
N = matrix of differential equation coefficients of command influence (implicit model)
T = discrete time step (sampling period)
u = vector of controls (inputs to the airplane control surfaces)
u_1,u_2,u_3 = elements of vector u
v_f = forward velocity
V = square symmetric positive semidefinite matrix (usually diagonal) of control cost in discrete-time cost function
V_c = square symmetric positive semidefinite matrix (usually diagonal) of control cost in continuous cost function
$\hat{V}$ = modified V, equal to $\psi'(W^+ + G_{xx}{}^+)\psi + V$
$\hat{V}_c$ = modified V_c, equal to $D'W_cD + V_c$
W = square symmetric positive semidefinite matrix (usually diagonal) of state cost in discrete-time cost function
W_c = square symmetric positive semidefinite matrix (usually diagonal) of state cost in continuous cost function
x = state vector of the airplane
$\dot{x}$ = time rate of change of x
$\dot{x}_m$ = desired $\dot{x}$ (in the implicit model)
y_1,y_2,y_3 = variables used to model control surface actuators
δ = vector of pilot's commands
$\theta,\dot{\theta}$ = pitch angle, rate, respectively
ϕ = state transition matrix corresponding to matrix A
ϕ_L = state transition matrix corresponding to matrix L
ψ = control transition matrix corresponding to matrix D
ψ_N = command transition matrix corresponding to matrix N

Superscripts

$(.)'$ = denotes transposition of $(.)$, a transposed vector is a row vector
$(.)^+$ = in discrete time equations, denotes the value of $(.)$ at time $t+T$, the same symbol without a superscript denotes the value of $(.)$ at time t.
$(.)^{-1}$ = denotes inverse of matrix $(.)$

Introduction

THE applications of optimal control theory to the design of airplane control systems have increased considerably in recent years. These techniques have well-recognized value in the synthesis problem because they achieve direct synthesis rather than synthesis through repeated trial and error. Furthermore, optimization theory furnishes both the gains and the structure of the control system.

Historically, airplane stability augmentation systems have been designed to improve stability and control deficiencies of unaugmented airplanes which the pilot deemed unpleasant or dangerous. The criteria to which the systems have been designed are often specified as acceptable handling quality regions for the poles and zeros of the unaugmented airplane transfer functions. In addition, classical performance criteria such as overshoot, time to half amplitude, response time, etc. have been specified. However, such criteria have been found increasingly inadequate as a basis for augmentation systems design for advanced aircraft.

In contrast to conventional control system design, optimal control is based upon the minimization of a cost function, subject to the constraint of the equations of motion of the system which is to be controlled. Although various forms of cost functions have been used in the past, (e.g., minimum time, minimum fuel, etc.), the quadratic cost function has been found to be most useful in the design of airplane control systems.[1–4] The general form of a quadratic cost function has been found most acceptable because for linear systems there exists an analytical solution which may be solved readily with comparatively modest digital computer capacity. Furthermore, the control is linear and the method is readily applicable to multivariable systems.

The particular form of the quadratic cost function, however, has been under study by a number of investigators. The optimum state regulator problem uses the state vector and the control vector in the cost function and has been discussed extensively in Refs. 5 and 6. The optimum output regulator problem replaces the state vector in the cost function by the output vector.[1,2]

The optimum state tracking problem has been formulated

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in Ref. 6. In this problem, the error between the state vector and the command input vector appears in the cost function and is minimized. The optimal output tracking problem[5] results in a system with output tracking in response to input commands and optimal output regulation in the absence of input commands. The basic difference between the optimal state tracking and optimal output tracking problems is in the error to be minimized. In the state tracking formulation, the error is defined as the difference between the state vector and the command vector. In the output tracking formulation, the error between the output vector and the command vector is minimized.

To achieve the real design objectives more adequately, the model-following optimization techniques were developed.[6–8] These techniques employ the desired closed-loop aircraft response in terms of a set of state equations representing a a model vehicle.

The model-in-the-system or explicit model-following method[13] uses the desired model in the control system as a prefilter ahead of the airplane. The optimal control employs feedforward and feedback gains such that the augmented airplane will follow the output of the model. The model-in-the-cost function or implicit model-following method uses optimal feedback gains to modify the unaugmented airplane characteristics such that they approach the model characteristics. In this method, the model is incorporated into the cost function.

Previous optimal model-following methods have been based upon the uncontrolled model dynamics only. This has led to difficulties, when these techniques are applied to system synthesis for desirable airplane handling qualities. It is well known that airplane handling qualities[9] are not only determined by the uncontrolled airplane dynamics, but also by the airplane's control characteristics. In order to allow for control effects in the model, attempts have been made to include the desirable control characteristics by adding command state equations into the model dynamics.[8] This approach also has not always proved satisfactory.

This paper extends the implicit model-following technique to include desirable control characteristics in the airplane model. Equations for the continuous optimal control system, as well as for the discrete optimal controller, are derived. The usefulness of this new technique is shown by applying it to the design of the controller for a STOL transport.

Airplane Model Definition for Desirable Handling Qualities

Most of the existing airplane flying qualities criteria are specified in government or service specifications.[10,11] They are written in terms of parameters which describe the stability and control of the conventional unaugmented airplane. Acceptable ranges are defined for the short period and phugoid responses in the longitudinal axes and for the Dutch roll response and roll and spiral time constants in the lateral-directional axes.

The effects of the control characteristics on the longitudinal flying qualities are accounted for by including normal acceleration response to stick deflection in the criteria. The desirable airplane responses to lateral control inputs are specified in the criteria for roll rate oscillations and roll rate requirements.

Any stability and control augmentation system which is added to an airplane exhibiting undesirable or unacceptable flying qualities has to be designed such that the dynamic responses of the augmented airplane match the desirable responses of the unaugmented airplane. Specification of airplane flying qualities in this manner is often very undesirable because stability augmentation system dynamics are important in the augmented airplane response and a comparison with the desirable unaugmented airplane is no longer possible. In addition, none of the presently existing specifications is in a form suitable for direct application to modern optimal control techniques in systems synthesis.

In order to design the control system for desirable closed-loop dynamics, as well as for good response to pilot commands, the following model equation is selected:

$$\dot{x}_m = Lx + N\delta \tag{1}$$

The vector x represents any airplane state vector for which the augmentation system is to be designed. The pilot command signals are the elements of the vector δ. The matrix L contains the desired characteristics without pilot commands. Desirable command characteristics are specified in the matrix N.

Because established criteria suitable for use in Eq. (1) are not available, the L matrix is chosen such that all state variables are decoupled and have a first order, exponential response to a step input. The elements of the matrix N are selected such that the pilot commands the desired state variables only.

Implicit Model Following

In the implicit model-following approach to optimal control design, the difference between actual and desired closed-loop equations is made part of the cost function to be minimized. The modeling is implicit in the sense that the response is forced to be close to the specified model, without actually performing on-board model calculations. Figure 1 shows the block diagram of an airplane control system design by the implicit model-following technique. The matrices A and D describe the given unaugmented airplane stability and control characteristics, respectively. The feedback matrix M and the feedforward matrix F are both determined from the implicit model control system synthesis. Their derivation is presented in the following section.

Derivation of Controller for Desired Handling Qualities

The airplane and control surfaces are represented by the vector differential equation (derived from Ref. 12)

$$\dot{x} = Ax + Du \tag{2}$$

The desired performance is specified by the implicit model, Eq. (1).

The approach used is to first assume a discrete time (sampled-data) controller. Later, the sampling period will be allowed to approach zero to obtain the optimal continuous controller. First, Eqs. (1) and (2) are converted to discrete transition equations. This is done by using Fath's method [14] to diagonalize A and L, finding the diagonalized transition matrices (see Ref. 15), and then reversing the diagonalization. Equation (1) becomes

$$x_m^+ = \phi_L x + \psi_N \delta \tag{3}$$

while Eq. (2) becomes

$$x^+ = \phi x + \psi u \tag{4}$$

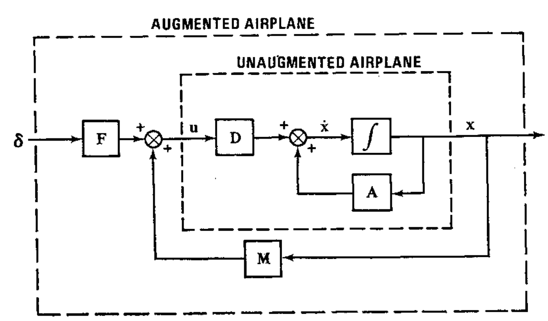

Fig. 1 System block diagram.

The problem is then to find feedback matrix M and feedforward matrix F, with

$$u = Mx + F\delta \tag{5}$$

so as to minimize the cost function

$$J = \Sigma(x^+ - x_m^+)'W^+(x^+ - x_m^+) + u'Vu \tag{6}$$

The summation in Eq. (6) is taken over enough time steps to make M and F constant over the first few time steps.

Applying the fundamental relations of discrete dynamic programming (see Ref. 16), the optimal control is determined backward in time. At any step,

$$J = (x^+ - x_m^+)'W^+(x^+ - x_m^+) + u'Vu + J^+ \tag{7}$$

At the last time step, J^+ is zero. Substituting Eqs. (3) and (4) into Eq. (7), with J^+ zero, expanding, and collecting terms, shows that J^+ in general takes the form

$$J^+ = x^{+\prime}G_{xx}^+x^+ - x^{+\prime}G_{x\delta}^+\delta^+ + \delta^{+\prime}G_{\delta\delta}^+\delta^+ \tag{8}$$

where G_{xx}^+ is symmetric. Substituting Eqs. (3, 4 and 8) into Eq. (7), expanding, differentiating with respect to u, and setting this derivative equal to zero, gives

$$\begin{aligned}[\psi'(W^+ + G_{xx}^+)\psi + V]u \\ = -\psi'[W^+(\phi - \phi_L) + G_{xx}^+\phi]x + \\ \psi'W^+\psi_N\delta + \tfrac{1}{2}\psi'G_{x\delta}^+\delta^+\end{aligned} \tag{9}$$

It is here assumed that $\delta^+ = \delta$. Two interpretations of this assumption are possible. First, it may be considered equivalent to assuming that changes in δ occur slowly. This is reasonable when it is observed that model responses are often derived for fixed δ after an initial step. There is also a stochastic viewpoint. Note that in actual operation, a discrete controller must compute u from x and δ, without any definite knowledge of δ^+. In this sense, assuming $\delta^+ = \delta$ is equivalent to assuming that changes in δ are uncorrelated with present or past values of δ, so that $\delta^+ = \delta$ is the best estimate of δ^+. The stochastic viewpoint for a continuous controller is that the best estimate for $\dot{\delta}$ is zero

Let

$$\hat{V} = \psi'(W^+ + G_{xx}^+)\psi + V \tag{10}$$

Then

$$M = -\hat{V}^{-1}\psi'[W^+(\phi - \phi_L) + G_{xx}^+\phi] \tag{11}$$

$$F = \hat{V}^{-1}\psi'[W^+\psi_N + \tfrac{1}{2}G_{x\delta}^+] \tag{12}$$

Substituting Eqs. (3–5, 8, 11 and 12) in Eq. (7) gives

$$J = x'G_{xx}x - x'G_{x\delta}\delta + \delta'G_{\delta\delta}\delta \tag{13}$$

with

$$G_{xx} = (\phi - \phi_L)'W^+(\phi - \phi_L + \psi M) + \phi'G_{xx}^+(\phi + \psi M) \tag{14}$$

$$\begin{aligned}G_{x\delta} = (\phi - \phi_L)'W^+(2\psi_N - \psi F) + (\psi M)'W^+\psi_N - \\ \phi'G_{xx}^+\psi F + (\phi + \tfrac{1}{2}\psi M)'G_{x\delta}^+\end{aligned} \tag{15}$$

$$G_{\delta\delta} = \psi_N'W^+(\psi_N - \psi F) - \tfrac{1}{2}F'\psi'G_{x\delta}^+ + G_{\delta\delta}^+ \tag{16}$$

If Eq. (11) is substituted into Eq. (14), it becomes apparent that G_{xx} remains symmetric.

Note that Eqs. (11) and (14) may be iterated independently of Eqs. (12, 15 and 16) and that Eqs. (11) and (14) do not contain ψ_N. This shows that the optimal feedback M is independent of matrix N.

At steady state, $G_{xx}^+ = G_{xx}$, $G_{x\delta}^+ = G_{x\delta}$. Solving Eq. (15) for $G_{x\delta}$, and substituting this solution in Eq. (12) yields, after some manipulation

$$F = \hat{V}^{-1}\psi'[I - (\phi - \psi M)']^{-1}(I - \phi_L')W\psi_N \tag{17}$$

Thus, if only the steady-state value of F is required, Eq. (17) may be used rather than iterating Eqs. (12) and (15).

For the continuous controller, the cost function is

$$J_c = \int_0^\infty [(\dot{x} - \dot{x}_m)'W_c(\dot{x} - \dot{x}_m) + u'V_cu]dt \tag{18}$$

A comparison of Eqs. (6) and (18), similar to that in Ref. 16, shows that

$$W = W_c/T \tag{19}$$

$$V = V_cT \tag{20}$$

As T approaches zero, ϕ approaches $I + AT$, ϕ_L approaches $I + LT$, ψ approaches DT, and ψ_N approaches NT. Substituting Eqs. (19) and (20) into Eqs. (11, 12, 14, 15 and 17), letting T approach zero, yields, after some manipulation,

$$M = -\hat{V}_c^{-1}D'[W_c(A - L) + G_{xx}] \tag{21}$$

$$F = \hat{V}_c^{-1}D'[W_cN + \tfrac{1}{2}G_{x\delta}] \tag{22}$$

$$\begin{aligned}-\dot{G}_{xx} = G_{xx}[A - D\hat{V}_c^{-1}D'W_c(A - L)] + \\ [A - D\hat{V}_c^{-1}D'W_c(A - L)]'G_{xx} + \\ (A - L)'(W_c - W_cD\hat{V}_c^{-1}D'W_c)(A - L) - \\ G_{xx}D\hat{V}_c^{-1}D'G_{xx}\end{aligned} \tag{23}$$

$$\begin{aligned}-\dot{G}_{x\delta} = 2(A - L)'[W_c - W_cD\hat{V}_c^{-1}D'W_c]N - \\ 2G_{xx}D\hat{V}_c^{-1}D'W_cN + (A + DM)'G_{x\delta}\end{aligned} \tag{24}$$

$$F = \hat{V}_c^{-1}D'[(A + DM)']^{-1}L'W_cN \tag{25}$$

where

$$\hat{V}_c = D'W_cD + V_c \tag{26}$$

and where Eq. (25), like Eq. (17), is used to find the steady-state value of F only. Eqs. (21) and (23) are exactly the same as those published previously by Gaul et al.,[6] Tyler[7] and Markland,[8] using the model

$$\dot{x}_m = Lx \tag{27}$$

Application to Control System Design

From a practical standpoint, the design computations derived previously must be performed by a digital computer. A computer program written for this purpose is described below. The design of an optimal control for a short takeoff and landing (STOL) aircraft is then given.

Description of Computer Program

A program has been written for the CDC 6600 digital computer to perform optimal control design. This program has the following features:

a) Inputs are made as convenient for the designer as possible. Inputs include aerodynamic and structural matrices, lift growth coefficients, actuator transfer functions, functions to shape the gust power spectral densities, model matrices, matrices of cost coefficients (W and V), and variances of gusts and measurement errors.

b) The program computes both continuous and discrete-time optimal state estimators and control gains. The computations for continuous control make use of Fath's method,[14] which achieves very short running time by avoiding numerical integration. The results for continuous control are then used to start the computations for discrete control, at a point fairly close to steady state, thus achieving short over-all running time.

c) System performance is evaluated by computing the variances of the estimation errors, states, and controls, and by computing selected frequency responses.

d) If the control must function at more than one flight condition, the program computes the best fixed control over the given set of flight conditions, using Stineman's method.[17]

Example of STOL Aircraft Control

The design methods of this paper have been applied to landing control of an STOL airplane. The airplane state vector is

$$x = [v_f \; \dot{h} \; \theta \; \dot{\theta} \; y_1 \; y_2 \; y_3]' \tag{28}$$

and the control vector is

$$u = [u_1 \; u_2 \; u_3]' \tag{29}$$

where u_1, u_2, and u_3 are inputs to the elevator, throttle, and spoiler actuators, respectively, and y_1, y_2 and y_3 are states used to model first-order lags for the actuators. The airplane characteristics are given by the matrices

$$A = \begin{bmatrix} -0.065 & -0.1078 & -15.47 & 0 & 0 & 61.4 & 5.735 \\ -0.434 & -0.57 & -88.4 & 0.391 & -99.82 & 0 & -169.1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.00072 & -0.00418 & -0.648 & -1.657 & -29.08 & 0 & -2.836 \\ 0 & 0 & 0 & 0 & -14.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -14.0 \end{bmatrix} \tag{30}$$

and

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

For the example being considered, it was desired that stick deflection command vertical velocity without affecting forward velocity, and that throttle deflection command forward velocity without affecting vertical velocity. Since forward velocity is nearly constant during landing, it is equivalent (with a simple scale change) to say that stick deflection commands flight path angle. The following model has the desired characteristics:

$$L = \begin{bmatrix} -0.2 & 0 & 0 & 0 & 0 & 61.4 & 5.735 \\ 0 & -2.0 & 0 & 0 & -99.82 & 0 & -169.1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & -29.08 & 0 & -2.836 \\ 0 & 0 & 0 & 0 & -14.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -14.0 \end{bmatrix} \tag{32}$$

$$N = \begin{bmatrix} 0 & -15.5 \\ -155 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{33}$$

This model has the desired decoupling of forward and vertical velocities. Note that the effects of the controls are modeled in L in exactly the same way as in A. In other words, the model does not attempt to change the control or actuator characteristics.

The design obtained depends not only upon the model but also upon the cost coefficients assigned to various parts of the model. The first four elements on the main diagonal of W are 10, 10, 0, and 1, respectively, with all other elements of W zero. This attaches high cost to failure to match the model equations for forward and vertical velocity. All other costs are low or zero. It may be observed that the model response to a step command includes zero pitch angle and pitch rate. However since no cost is attached to variations in pitch angle, the optimal design will automatically select the pitch angle that gives the best match to the desired forward and vertical velocity. The small cost attached to pitch rate tends to prevent extremely high pitch rates, but otherwise allows essentially whatever pitch rate is best for achieving the desired velocities.

The cost coefficients in matrix V were chosen to limit the amount of feedback through matrix M. To do this, horizontal and vertical gust effects were added to the equations, with gust power spectral densities taken from Ref. 10. The coefficients in V were then chosen by trial to give the following probable control surface action due to feedback through M:

Control	*Deflection* (1σ)
Elevator	5.7°
Throttle	10.9%
Spoiler	5.1°

The time response to a step command tends to fall short of the desired response for two reasons. First, the cost function contains the slopes of the states, rather than the states themselves. Hence, steady-state errors do not contribute directly to cost. Second, the cost attached to control effort, needed to limit M, also tends to reduce the feedforward matrix F.

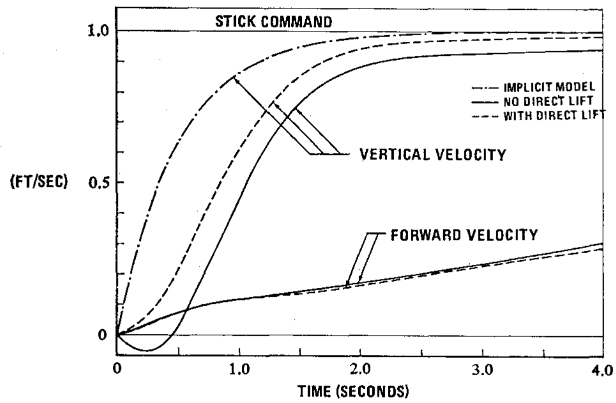

Fig. 2 Model and airplane velocity responses to step stick command.

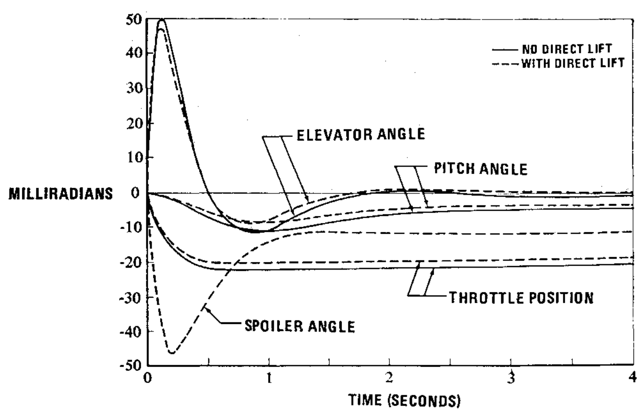

Fig. 3 Pitch and control responses corresponding to Fig. 2.

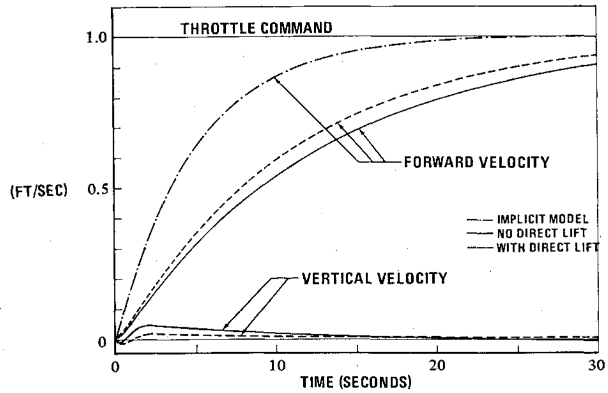

Fig. 4 Model and airplane velocity responses to step throttle command.

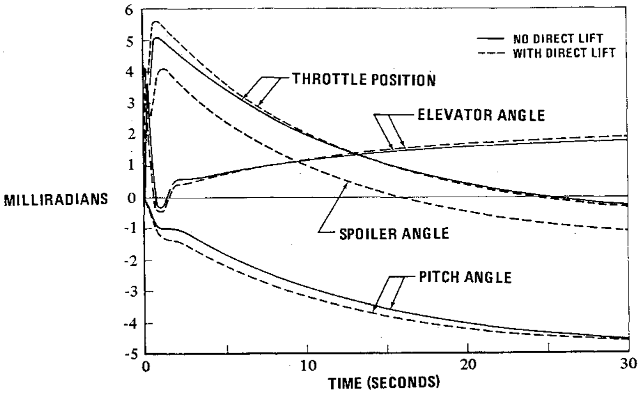

Fig. 5 Pitch and control responses corresponding to Fig. 4.

However, these factors do not affect the shape of the time response, so the problem is easily overcome by making small scale changes after observing the time responses.

Time responses are shown in Figs. 2 and 3 for a step stick displacement, and in Figs. 4 and 5 for a step throttle displacement. The figures show the improvement resulting from the use of direct lift control. Whether or not the improvement justifies the added complexity is left to the reader's judgment.

Conclusions

The technique of implicit model-following[6–8] may be extended to include the desired effects of pilot commands. This provides a convenient means for including all handling quality requirements in the control synthesis. That is, the extended implicit model includes not only the desired stability and damping for unforced response, but also includes the desired response to pilot commands.

A longitudinal implicit model has been derived such that all state variables are decoupled, and the desired airplane responses to pilot inputs are easily specified.

The derivation of the optimal control with implicit model-following shows that the optimal state feedback matrix is independent of the desired response to pilot commands. Therefore, the equations for computing the feedback matrix agree with previous derivations.[6–8] The optimal feed-forward matrix from pilot commands to the controls depends upon all inputs to the problem.

A digital computer program has been prepared to execute the design computations for optimal control. In the example considered, this program was used to design the optimal implicit-model-following control for an STOL airplane. In this design, it was desired that vertical velocity be commanded by stick position, that changes in stick position have no effect on forward velocity, and that throttle changes have no effect on vertical velocity. A model having these characteristics was derived. The computed airplane responses using the optimal control, were found to agree closely with the model.

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